

## Selective Search in Games of Different Complexity



# Selective Search in Games of Different Complexity

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# Table of Contents

<b>Preface</b>	<b>v</b>
<b>Table of Contents</b>	<b>vii</b>
<b>List of Figures</b>	<b>xi</b>
<b>List of Tables</b>	<b>xii</b>
<b>List of Algorithms</b>	<b>xiii</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Games . . . . .	1
1.1.1 Game Theory . . . . .	2
1.1.2 Games and AI . . . . .	3
1.2 Selective Search . . . . .	5
1.3 Problem Statement and Research Questions . . . . .	7
1.4 Thesis Overview . . . . .	8
<b>2 Search Methods</b>	<b>11</b>
2.1 Searching in Games . . . . .	11
2.2 Minimax . . . . .	12
2.3 $\alpha\beta$ Search . . . . .	13
2.4 Move Ordering . . . . .	15
2.4.1 Killer Heuristic . . . . .	15
2.4.2 History Heuristic . . . . .	16
2.5 Iterative Deepening . . . . .	16
2.6 Transpositions . . . . .	17
2.6.1 Transposition Tables . . . . .	17
2.6.2 Enhanced Transposition Cutoff . . . . .	18
2.7 Monte-Carlo Tree Search . . . . .	18
2.7.1 Structure of MCTS . . . . .	19
2.7.2 Enhancements . . . . .	21
2.7.3 Parallelization . . . . .	22

<b>3</b>	<b>Single-Player Monte-Carlo Tree Search</b>	<b>25</b>
3.1	SameGame . . . . .	26
3.1.1	Rules . . . . .	26
3.1.2	Complexity of SameGame . . . . .	27
3.1.3	Related Work . . . . .	28
3.2	A* and IDA* . . . . .	29
3.3	Single-Player Monte-Carlo Tree Search . . . . .	30
3.3.1	Selection Step . . . . .	30
3.3.2	Play-Out Step . . . . .	31
3.3.3	Expansion Step . . . . .	32
3.3.4	Backpropagation Step . . . . .	32
3.3.5	Final Move Selection . . . . .	32
3.3.6	Randomized Restarts . . . . .	32
3.4	The Cross-Entropy Method . . . . .	33
3.5	Experiments and Results . . . . .	34
3.5.1	Simulation Strategy . . . . .	34
3.5.2	Manual Parameter Tuning . . . . .	34
3.5.3	Randomized Restarts . . . . .	36
3.5.4	Time Control . . . . .	36
3.5.5	CEM Parameter Tuning . . . . .	37
3.5.6	Comparison on the Standardized Test Set . . . . .	38
3.6	Chapter Conclusions and Future Research . . . . .	40
<b>4</b>	<b>Proof-Number Search with Endgame Databases</b>	<b>41</b>
4.1	Solving Games . . . . .	42
4.2	Fanorona . . . . .	43
4.2.1	Board . . . . .	43
4.2.2	Movement . . . . .	44
4.2.3	End of the Game . . . . .	46
4.3	Analyzing Fanorona . . . . .	46
4.4	Retrograde Analysis . . . . .	50
4.5	Proof-Number Search . . . . .	52
4.5.1	PN Search . . . . .	52
4.5.2	PN <sup>2</sup> Search . . . . .	53
4.6	Experiments and Results . . . . .	53
4.6.1	Solving Fanorona and its Smaller Variants . . . . .	53
4.6.2	Tradeoff between Backward and Forward Search . . . . .	56
4.6.3	Behavior of the PN Search . . . . .	58
4.7	Correctness . . . . .	58
4.8	Chapter Conclusions and Future Research . . . . .	60
<b>5</b>	<b>Forward Pruning in Chance Nodes</b>	<b>63</b>
5.1	Expectimax . . . . .	64
5.2	Pruning in Chance Nodes . . . . .	66
5.2.1	Star1 . . . . .	66
5.2.2	Star2 . . . . .	67

5.3	Forward Pruning . . . . .	69
5.4	ChanceProbCut . . . . .	70
5.5	Test Domain . . . . .	72
5.5.1	Stratego . . . . .	72
5.5.2	Dice . . . . .	76
5.5.3	ChanceBreakthrough . . . . .	76
5.5.4	Game Engines . . . . .	77
5.6	Experiments and Results . . . . .	79
5.6.1	Stratego . . . . .	79
5.6.2	Dice . . . . .	82
5.6.3	ChanceBreakthrough . . . . .	85
5.7	Chapter Conclusions and Future Research . . . . .	88
<b>6</b>	<b>Best-Reply Search in Multi-Player Games</b>	<b>89</b>
6.1	Coalition Forming in Multi-Player Games . . . . .	90
6.2	Search Algorithms for Multi-Player Games . . . . .	91
6.2.1	Max <sup>n</sup> . . . . .	91
6.2.2	Paranoid . . . . .	92
6.3	Best-Reply Search . . . . .	93
6.3.1	Idea . . . . .	94
6.3.2	Pseudo Code . . . . .	95
6.3.3	Best-Case Analysis of BRS . . . . .	95
6.3.4	Strengths and Weaknesses of BRS . . . . .	96
6.4	Test Domain . . . . .	96
6.4.1	Chinese Checkers . . . . .	97
6.4.2	Focus . . . . .	98
6.4.3	Rolit . . . . .	99
6.5	Experiments and Results . . . . .	101
6.5.1	Validation . . . . .	102
6.5.2	Average Search Depth . . . . .	102
6.5.3	BRS against Max <sup>n</sup> . . . . .	103
6.5.4	BRS against Paranoid . . . . .	104
6.5.5	BRS vs. Paranoid vs. Max <sup>n</sup> . . . . .	106
6.6	Chapter Conclusions and Future Research . . . . .	107
<b>7</b>	<b>Conclusions and Future Research</b>	<b>109</b>
7.1	Conclusions on the Research Questions . . . . .	109
7.1.1	One-Player Games . . . . .	110
7.1.2	Two-Player Games . . . . .	110
7.1.3	Two-Player Games with Non-Determinism and Imperfect In- formation . . . . .	111
7.1.4	Multi-Player Games . . . . .	112
7.2	Conclusion on the Problem Statement . . . . .	113
7.3	Recommendations for Future Research . . . . .	113
	<b>References</b>	<b>115</b>

<b>Index</b>	<b>137</b>
<b>Summary</b>	<b>139</b>
<b>Samenvatting</b>	<b>143</b>
<b>Curriculum Vitae</b>	<b>149</b>
<b>SIKS Dissertation Series</b>	<b>151</b>



## List of Figures

2.1	An example minimax tree. . . . .	13
2.2	An example $\alpha\beta$ tree. . . . .	15
2.3	The four steps of MCTS (slightly adapted from Chaslot <i>et al.</i> , 2008d). . . . .	20
3.1	Example SameGame moves. . . . .	27
3.2	The average score for different settings of randomized restarts. . . . .	36
4.1	The initial position of a Fanorona game. . . . .	44
4.2	Initial positions for the $5\times 7$ and $7\times 5$ board. . . . .	44
4.3	An example position during a Fanorona game. . . . .	45
4.4	Two positions without legal moves for one side encountered during the search. . . . .	47
4.5	Ratio of capturing moves and paika moves as a function of the number of pieces on the board. . . . .	48
4.6	The average branching factor as a function of the number of pieces on the board. . . . .	48
4.7	The average number of pieces as a function of the move number. . . . .	49
4.8	$3\times 3$ board: White can force a win. . . . .	55
4.9	Total time required for solving Fanorona variants. . . . .	58
4.10	Development of the proof and disproof number when proving that White cannot win the initial $5\times 9$ Fanorona position. . . . .	59
4.11	Development of the proof and disproof number when proving that White can at least draw the initial $5\times 9$ Fanorona position. . . . .	59
4.12	Development of the proof and disproof number when proving that White can win the initial position on the $7\times 3$ board. . . . .	59
5.1	An example expectimax tree. . . . .	65
5.2	Search windows at chance nodes cause incorrect pruning. . . . .	66
5.3	Successful Star1 pruning. . . . .	67
5.4	Successful Star2 pruning in the probing phase. . . . .	68
5.5	ChanceProbCut pruning. . . . .	71
5.6	The regular search prunes with help of ChanceProbCut. . . . .	72
5.7	A possible initial position in Stratego. . . . .	74
5.8	Initial position of ChanceBreakthrough. . . . .	77
5.9	Evaluation pairs at depths 2 and 4 in Stratego. . . . .	79
5.10	Evaluation pairs at depths 3 and 5 in Stratego. . . . .	80
5.11	Evaluation pairs at depths 3 and 7 in Dice. . . . .	83
5.12	Evaluation pairs at depths 5 and 9 in Dice. . . . .	83
5.13	Evaluation pairs at depths 1 and 3 in ChanceBreakthrough. . . . .	86
5.14	Evaluation pairs at depths 2 and 4 in ChanceBreakthrough. . . . .	86
5.15	Evaluation pairs at depths 3 and 5 in ChanceBreakthrough. . . . .	87
6.1	An example $\max^n$ tree. . . . .	91
6.2	An example paranoid tree. . . . .	93

6.3	An example BRS tree. . . . .	94
6.4	A three-player Chinese Checkers board. . . . .	97
6.5	Setups for Focus. . . . .	98
6.6	Example move for Focus. . . . .	99
6.7	Setups for Othello and Rolit. . . . .	100

## List of Tables

1.1	Games of different classifications. . . . .	5
3.1	Effectiveness of the simulation strategies. . . . .	34
3.2	Results of SP-MCTS for different settings. . . . .	35
3.3	Average score on 250 positions using different time control settings. . . . .	37
3.4	Parameter tuning by CEM. . . . .	37
3.5	Average scores of CEM tuning. . . . .	38
3.6	Comparing the scores on the standardized test set. . . . .	39
4.1	Estimated complexities with increasing database size. . . . .	49
4.2	Database sizes up to 15 pieces. . . . .	51
4.3	Number of won, drawn, and lost positions in the databases for the 5×9 board. . . . .	52
4.4	The game-theoretic values for different board sizes. . . . .	54
4.5	Effectiveness of endgame databases for 5×5 Fanorona. . . . .	56
4.6	Effectiveness of endgame databases for 7×5 Fanorona. . . . .	57
4.7	Effectiveness of endgame databases for 3×9 Fanorona. . . . .	57
5.1	Stratego piece values. . . . .	78
5.2	Performance of ChanceProbCut at depth 7 for 500 Stratego positions. . . . .	80
5.3	Performance of ChanceProbCut at depth 9 for 500 Stratego positions. . . . .	81
5.4	Performance of ChanceProbCut at depth 11 for 500 Stratego positions. . . . .	81
5.5	Stratego selfplay experiment, 1 second per move. . . . .	82
5.6	Performance of ChanceProbCut at depth 9 for 1,000 Dice positions. . . . .	84
5.7	Performance of ChanceProbCut at depth 11 for 1,000 Dice positions. . . . .	84
5.8	Performance of ChanceProbCut at depth 13 for 1,000 Dice positions. . . . .	84
5.9	11×11 Dice selfplay experiment, 100 ms per move. . . . .	85
5.10	Performance of ChanceProbCut at depth 5 for 500 ChanceBreakthrough positions. . . . .	87
5.11	ChanceBreakthrough selfplay experiment, 5 seconds per move. . . . .	88
6.1	Winning statistics for Paranoid vs. Max <sup>n</sup> for Chinese Checkers with 250 ms per move. . . . .	102
6.2	Winning statistics for Paranoid vs. BRS for two-players with 250 ms per move. . . . .	103
6.3	Average search depth. . . . .	104
6.4	Winning statistics for BRS vs. Max <sup>n</sup> . . . . .	105

6.5	Winning statistics for BRS vs. Paranoid. . . . .	106
6.6	Tournament results. . . . .	107

## List of Algorithms

2.1	Negamax pseudo code. . . . .	13
2.2	$\alpha\beta$ pseudo code. . . . .	14
5.1	$\alpha\beta$ search part of expectimax. . . . .	72
5.2	ChanceProbCut for chance nodes. . . . .	73
6.1	Best-Reply Search. . . . .	95